Figure 2.9a Wave Drift Force and Motion for Semisubmersibles (Beam Seas)
Figure 2.9b Wave Drift Force and Motion for Semisubmersibles (Bow Seas)
conservative answers. But due to the difficulty in predicting the magnitude of resulting low frequency tensions, their effect is neglected.

2.3.4 Verification of the Simplified Load Model

The procedure used to estimate the wave forces acting on MODUs has been verified and calibrated against results from more sophisticated computer programs. In an initial verification effort, the computer output for one design wave cases on single surface piercing cylindrical piles were used. These data were produced during an analytical wave force study conducted by Exxon and Shell Research Companies and documented by Bea (1973). In this study, the maximum wave force acting on a 20 ft diameter surface piercing cylinder was estimated where nondimensional water depths \( d/gT^2 \) is 0.028. Based on the simplified procedure developed in the previous sections of this chapter, the maximum wave force acting the same cylinder is also estimated using Stokes fifth-order and depth stretched linear wave theories. A inertia coefficient of \( C_w=1.5 \) is used. The results are also compared to those gained by using Dean’s Charts that are developed based on ninth-order stream function theory (Dean, 1973). The results are summarized in Figure 2.10.

From Figure 2.10 it can be found that, Stokes V results in an estimate of base shear which is in good agreement with results reported in Exxon-Shell wave force study, which is about 1900 kips. Dean’s Charts result, which is about 1600 kips, slightly underpredict the total force. Surprising is the result gained by using depth-stretched linear wave theory, which gives a base shear that is almost 40% less than that given by Stokes V.
Field measurements in intermediate water depths indicate that depth-stretched Airy theory provides an acceptable fit to the actual wave kinematics. With this in mind, the results plotted in Figures 2.10 indicate that wave force predictions based on finite amplitude wave theories (Stokes V or stream function) might be conservatively biased. The variabilities of the force coefficients given by Bea (1990) were used to estimate the uncertainties associated with the wave force, which were $B = 0.66$ and $V_s = 0.47$. These estimates are consistent with the simplified analytical models employed to calculate the loadings. Based on the results of this initial verification case study, Stokes fifth-order theory was used in this research.

![Comparison of Stokes and Airy forces](image)

Figure 2.10: Wave Force on a Vertical Surface Piercing Cylinder in Transitional Water ($d / gT^2 = 0.028$)
2.4 MODU Mooring Capacity

2.4.1 Overview of Mooring Analysis

Generally, the mooring system of a semi-submersible is as shown in Figure 2.11. Permanent mooring systems should be designed for two primary considerations: system overloading and fatigue. For MODU moorings, only analysis for extreme response is required.

Figure 2.11 Mooring Configuration
Extreme responses normally govern the design of the MODU mooring. They include MODU offset, mooring line tension, anchor load, and suspended line length. The environmental effects can be divided into three categories:

- Steady state forces including current force, mean wind and mean wave drift forces;
- Low frequency MODU motions due to wind and waves;
- Wave frequency MODU motions.

The responses of a mooring system to mean forces are predicted by static catenary equations. Generally speaking, the responses to low frequency motions can also be predicted by the same method because of the long periods of these motions. The responses to wave frequency MODU motions are usually predicted by one of the following two methods:

(1) Quasi-Static Analysis

In this approach, the dynamic wave loads are taken into account by statically offsetting the MODU by an appropriately defined wave included motion. MODU moored motions and dynamic effects associated with mass, damping and fluid acceleration are neglected. Research in mooring line dynamics has shown that the reliability of the mooring designs based on this method can vary widely depending on the MODU type, water depth and line configuration. Therefore, the quasi-static method is not recommended for the final design
of a permanent mooring. However, because of its simplicity, this method can be used for temporary moorings and preliminary studies of permanent moorings with higher factors of safety.

(2) Dynamic Analysis

Dynamic analysis accounts for the time-varying effects due to mass, damping, and fluid acceleration. In this approach, the time-varying fairlead motions are calculated from the MODU's surge, sway, heave, pitch, roll and yaw motions. Generally it is sufficient to account for only the vertical and horizontal fairlead motions in the plane of the mooring line. Dynamic models are used to predict mooring line responses to the fairlead motions. Several dynamic analysis techniques are available. The distinguishing feature among various dynamic analysis techniques is the degree to which non-linearity are treated. There are four primary nonlinear effects which can have an important influence on mooring line behavior:

- Nonlinear Stretching Behavior of the Line
- Changes in Geometry
- Fluid Loading
- Bottom effects

Two methods, frequency domain and time domain analyses, are commonly used for predicting dynamic mooring loads. In the time domain method, all of the nonlinear effects
can be modeled. The elastic stretch is mathematically modeled, the full Merinos equation
is included, the position of the mooring line is updated at each time step and the bottom
interaction is included using a frictional model. The general analysis implies the
recalculation of each mass term, damping term, stiffness term, and load at each time step.
Hence, the computation can become complex and time consuming.

The frequency domain method, on the other hand, is always linear because the principle of
linear superposition is used. Hence, all nonlinearities must be eliminated, either by direct
linearization or by an iterative linearization.

The procedure outlined below is recommended for the analysis of extreme response using
a quasi-static or dynamic approach. The calculated response in accordance with this
procedure should satisfy the design criteria.

The analysis is normally performed with the following computer programs:

1) Hydrodynamic Motion Analysis programs

These programs are used to determine wave frequency and low frequency vessel motions.

2) Static Mooring Analysis program

This program is used to analyze mooring line response to steady state environmental
forces and low frequency motions.

3) Dynamic Mooring Analysis program

This program is used to analyze mooring line response to wave frequency motions.
The recommended analysis procedure is described below (API, 1994):

a) Determine wind and current velocities, and significant wave heights and periods, for both the maximum design, and operating conditions in accordance with guidelines.

b) Determine the mooring pattern, characteristics of chain and wire rope to be deployed, and initial tension.

c) Determine the steady state environmental forces acting on the hull.

d) Determine the vessel's mean offset due to the steady state environmental forces using the static mooring analysis program.

e) Determine the low frequency motions. Since calculation of low frequency motions requires the knowledge of the mooring stiffness, the mooring stiffness at the mean offset should be determined first using a static mooring analysis computer program.

f) Determine the significant and maximum single amplitude wave frequency vessel motions using a hydrodynamic motion analysis program.

g) Determine the vessel's maximum offset, suspended line length, quasi-static tension, and anchor load.

h) Determine the maximum line tension and anchor load. A frequency domain or time domain dynamic mooring analysis program should be used.

i) Compare the maximum vessel offset and suspended line length from step g and maximum line tension and anchor load from step g or h. If the criteria are not met, modify the mooring design and repeat the analysis.
2.4.2 Mooring Analysis in MODUSIM

Due to the complexity of the mooring analysis procedures, it is difficult, if not impossible, to perform detailed mooring analysis procedures in each simulation step. That would be both time consuming and unnecessary. A simplified mooring capacity model is developed for the simulation purpose.

In MODUSIM, it is assumed that the total expected lateral capacity of the mooring system is $\overline{R_s}$ (Figure 2.11). There are two modes of mooring system failure:

1) all the mooring lines are broken and the MODU is in the free floating condition;

2) the horizontal hurricane load is larger than the total anchor holding force and some of the mooring lines are not broken so that the MODU is in a dragging condition. Thus:

$\overline{R_{a1}}$, failure mode one (free floating), and

$\overline{R_{a2}}$, failure mode two (anchors dragging).

These failure modes are determined by the MODUSIM user. The effects of different mooring line models are not considered. Users can modify $\overline{R_s}$ to include such effects.

For more detailed mooring analysis in MODUSIM, users can use the following simplified formulation which is derived from regression analysis to determine the maximum line tension of the MODU in different environmental conditions:

1. Determine mean environmental force;

2. Determine mean offset;

\[
\text{Mean Offset} = A \cdot F_{\text{mean}} + B \quad (2.51)
\]
3. Determine dynamic offset;

\[ \text{Dyn. Offset} = C \times H_s^2 + D \times H_s + E \]  \hspace{1cm} (2.52)

Where \( H_s \) is the significant wave height.

4. Determine total offset;

\[ \text{Total Offset} = \text{Mean Offset} + \text{Dyn. Offset} \]  \hspace{1cm} (2.53)

5. Determine maximum line tension.

\[ \text{Tension} = F \times \text{Tot. Offset}^2 + G \times \text{Tot. Offset} + H \]  \hspace{1cm} (2.54)

Parameters \( A, B, C, D, E, F, G, H \) are determined from regression analysis by users.

If information on the parameters are not available, it is recommended to use the assumption that if the environmental force is larger than the mooring capacity, the mooring system will fail.

2.4.3 Water Depth Factor

The algorithm discussed above is specific for one rig type at a given water depth and mooring system. The maximum line tension calculated from the processes presented above is a function of vessel type, mooring system, mean offset wave height and water depth. Changes in any of these make the constants in the algorithm change. The influence of these different variables can cause havoc when trying to create a simple algorithm. For example, a given dynamic offset may increase tensions significantly in shallow water and have no impact in deep water. With a large mean offset, a small dynamic offset can cause a large increase in tensions. The variables are highly non-linear and difficult to predict.
When determining a safe location to stack a MODU, one of the important criteria is adequate water depth. To have the best possible chance for survival in a hurricane, this may be interpreted as choosing a location with the optimal water depth for the mooring system. Research results from Noble Denton Inc. shows that as a rig is moved into shallow water, the capacity of the mooring system decreases (Noble Denton, 1991). This reflects an increase in mooring system stiffness as water depth decreases, and for a stiffer system, a given vessel offset will produce larger tensions.

A possible simple way to solve this problem is to introduce a "water depth correction factor" to modify the calculated forces. The forces are calculated as before and then multiplied by a "water depth correction factor". The resulting force could then be used to determine the approximate total tension. Based on the mooring system performance curves from Noble Denton (Noble Denton, 1994), a regression analysis was performed and the water depth correction factor (WDF) was defined as (Figure 2.12):

\[
WDF = 0.056 \left( \frac{\text{water depth}}{250} \right)^2 - 0.45 \left( \frac{\text{water depth}}{250} \right) + 1.89
\]

(2.55)

where the unit of water depth is feet.
2.5 Summary

A simulation procedure to characterize hurricane generated wind, current and wave fields with the consideration of shoaling effects was developed based on Cooper (1988). A simplified environmental loading model was developed that is able to develop estimates of total lateral wind, wave and current loadings acting on MODUs. Based on sustained wind velocity at a reference height, wind forces are estimated according to API RP 2A (API, 1993a). The wave loading prediction model utilizes Stokes fifth-order wave theory. The current velocity profile is added to the wave velocity profile. Wave directional spreading and current blockage are taken into account. The hydrodynamic forces acting on a simplified model of the structure is estimated using the Morison's equation.

The simplified load prediction procedure was verified with results reported in a wave force study performed by Exxon and Shell Research Companies (Bea, 1973). Good agreement
has been achieved for wave loading on a surface piercing cylinder in transitional water depth conditions using Stokes V theory.

Based on API (1994), a simplified MODU mooring capacity model was developed. The model takes account two mooring system failure modes: free floating and anchors dragging. A "water depth factor" was proposed to modify the calculated forces for different water depths.
CHAPTER 3

HURRICANE TRACK FORECASTING SIMULATION MODELS

3.1 Introduction to Hurricane Forecasting

The goal of hurricane forecasting is to predict accurately the temporal evolution of the areas of significant surface winds and heavy precipitation. The precipitation issue will not be considered here. The ideal hurricane wind-area prediction would accurately specify at any time the horizontal distributions of wind areas expected to contain damaging winds & cause high seas, perhaps in categories of minor, major and severe. From a user's perspective, the primary concern is the accuracy of forecast times of these wind speeds for a particular location. Forecasters usually think in terms of hurricane track, intensity, and size.

The track, intensity, and size components of a hurricane forecast are dynamically interdependent. For example, even if the intensity and size of a hurricane were to be precisely forecast, a relatively small cross-track forecast error might lead to an extreme overestimate of surface winds for a location near the forecast track. Conversely, an accurate intensity and track forecast accompanied by a size forecast that fails to account for substantial growth of the extent of gale-force winds will lead to a large underestimate in wind speed at a location forecast. Since this research deals with structures that are located on the continental shelf of the Gulf of Mexico, with water depth up to 600 ft, and the changes in the storm parameters after shelf-edge crossing are usually very small until
they reach land, the size and intensity of the hurricane are assumed to be stationary during passage over the continental shelf. The important factor remaining is the hurricane track forecast.

One of the simplest track prediction schemes is to assume persistence of recent motion. The physical basis for this assumption is that a tropical cyclone is normally a small vortex embedded in a large-scale flow. The recent motion of the storm is a result of the interaction of the vortex and the large-scale flow. If the vortex, large-scale flow and the interaction processes do not change, the future motion should resemble the past motion. Persistence is a reasonable, first-order approximation for prediction short-term motion.

The simplest prediction of the future track is to assume that the present storm will move with the average direction and speed of all past storms near that location. This is another important hurricane track prediction scheme, climatology. To make a track prediction, the climatological velocity vectors at the appropriate locations are multiplied by the time interval and the resulting displacements are added to the present latitude and longitude. The pure climatology forecast is more effective at longer forecast intervals.

A combination of persistence plus climatology is expected to provide an improvement over the separate techniques. A typical combination is half persistence and climatology. An alternative is a blend that weights persistence higher early in the forecast and
climatology higher later. This empirical blending presumably takes advantage of the best characteristics of each scheme.

Statistical track forecasting models derive their variance reducing potential from one or more of four sources of predictive information: climatology, persistence, environmental data and numerically forecast environmental data. A statistical combination of CLImatology and PERsistence (CLIPER), developed for the Atlantic region by Neumann (1972), has been extended to other basins (Leftwich and Neumann, 1977; Xu and Neumann, 1985). A similar persistence and climatology technique for the Western North Pacific is used by the Shanghai Typhoon Institute (Z. Wu, 1985, IWTC). Predictors such as the present latitude and longitude, the components of the recent motion of the storm and the intensity are used. Least-squares fitting of the basic predictors and various polynomial combinations are used in CLIPER to derive regression equations for future latitudinal/longitudinal displacements in 12-hour increments. Thus, this technique makes use of the “climatology” of past tropical cyclone tracks and the persistence components of the present storm to generate a forecast.

Several forecast centers also use the probabilities to describe uncertainties in the spatial and temporal occurrence of tropical cyclones. The most common uses of forecast probability in relation to the occurrence of severe cyclonic effects such as the distribution of hurricane-force winds, the height of sea waves and the elevation of storm surges, are:
a. to extend the usable length of forecasts despite their increasing uncertainty as the forecast period increases;
b. to provide a quantitative assessment of the threat posed by a cyclone approaching possible landfall;
c. to compare the relative threat to different places at the same time, or at different times as a threat develops;
d. to cause a consistent response to the same or similar set of circumstances; and
e. as a tool in risk analysis, both in respect to long-term protective measures, and for contemporary warning purposes.

For a comprehensive overview of the types of probabilities presently available, the reader is referred to Jarrel and Brand (1983).

3.2 Track Forecasting Models for Simulation

In previous simulation studies (Wen, 1988), the hurricane was assumed to be a storm traveling along a straight line with a given translation speed and direction. In fact, hurricane tracks are generally curved. Based on a statistical analysis of hurricane route histories, one can characterize the parameters that influence changes of hurricane direction, and estimate their probability distributions.

All the models discussed above are suitable for forecasting the track of a typical incoming hurricane. For a model to be suitable for Monte Carlo simulation, it must be complete
enough to include all the available hurricane information but still simple enough to run quickly if many simulations are needed. All the methodologies existing or discussed above are complicated and time consuming, and not suitable for simulation. As a result, two hurricane track forecasting model are developed especially for Monte Carlo simulation purposes:

1) **The Track Forecast Error Statistical Model** generates hurricane track forecast based on the 72 hour real-time hurricane forecast from hurricane forecast centers, and statistical analysis of historical hurricane track forecast error from the same hurricane forecast center. This model is especially useful to generate hurricane track forecast to perform simulations for a special incoming hurricane with 72 hour hurricane forecast.

2) **The Markov-Chain Simulation Model** generates hurricane track forecasts based on the Markov transition probability matrix, developed from hurricane track history statistics. This model is especially useful to generate hurricane track forecasts to perform simulations of long-term period, for example, one year, while the real time forecast data are not available during the period. This model is used in this study to calculate MODUs annual collision probabilities.

**3.3 Track Forecast Error Statistical Model**

Hurricane track actual and forecast data are obtained from the Joint Typhoon Warning Center (JTWC). For each past hurricane, the forecast track and actual track are compared
and forecast errors are calculated. Forecast errors are decomposed into along-track errors (ahead & behind) and cross-track errors (left & right). The errors are also decomposed into orientation and distance errors. As in Figure 3.1, the forecast distance error is defined as the distance between the forecast track point and the actual track point, and forecast orientation error is defined as the angle between the forecast track direction and the straight line passing through the forecast track point and actual track point. For a typical 72 hour hurricane track forecast, the data are given at 6 hour time intervals. The histogram of forecast errors are calculated on different forecast time interval data (6, 12, 24, 36, 48, and 72 hour intervals).

Figure 3.1 Determining the Track Forecast Error
Comparison between forecast error data in different sections and best-fitting Weibull and Uniform distributions are presented in Figure 3.2. Goodness-of-Fit tests (Chi-Square Test) were also performed for each fitted distributions (Refer to Appendix C for detailed procedures about these tests). It was found that, from the statistical point of view, adequacy of the fit varied considerably. For fit results of distance error, some of the fit results are good, e.g., for distance error of 36 hours and right error section, the significance level for the hypothesis that the fitted distribution gave the data is 4%. But most of the fit results have a large significance level and can not pass the goodness-of-fit test with significance level 5%, e.g., distance error of 24 hour right section.

Direction errors were analyzed similarly, to obtain distributions that characterize the input data adequately for the purpose of this engineering application.

Among the 8 distribution families fitted to the distance error data, a Weibull distribution was found to fit best. The best fitting distribution for direction error among these fittings is a uniform distribution. Also, it was found that the correlation between the orientation error and the distance error was insignificant.
Distance_Ahead_12 hour  Direction_Ahead_12 hour

Distance_Left_12 hour  Direction_Left_12 hour

Distance_Right_12 hour  Direction_Right_12 hour

Distance_Behind_12 hour  Direction_Behind_12 hour

Figure 3.2 Fit Results for Distance and Direction Forecasting Error
Distance_Ahead_36 hour

Direction_Ahead_36 hour

Distance_Left_36 hour

Direction_Left_36 hour

Distance_Right_36 hour

Direction_Right_36 hour

Distance_Behind_36 hour

Direction_Behind_36 hour
To perform simulations, a circular region of radius from 100-500 nm is divided into small sections of equal angular & radius extent. The angular interval is 30°; the radial interval is 30 nm (Figure 3.3). For example, for 24 hours interval forecast, the chance that the forecast distance error beyond a circle of radius 300 nm is negligible. Within the circular region, the chance that the storm center is in each small section at the end of the forecast interval can be estimated. The sum of all such probabilities must be 1.

![Diagram of probability function](image)

**Figure 3.3 Example of Calculation of Probability Function in a Square**

The procedure for estimating the probability function of the shaded square in Figure 3.3 for a 48 hour forecast point is as follows:

\[
P = P_{\text{left}} \cdot P_{\text{right}} \cdot P_{\text{behind}} (30 < \text{Error} < 60)
\]  

(3.1)
Where \( P_{\text{ase}} \) is the weighting function for each angular section (Table 3.1). For 48 hour forecast and the Left section here, \( P_{\text{ase}} = 0.2 \). \( P_{\text{de}} \) is the weighting function for direction error, here \( P_{\text{de}} = 1/3 \) because the direction errors are uniformly distributed and the shaded area has 30° angular extend, 1/3 of the Left section. \( P_{\text{omde}} (30 < \text{Error} < 60) \) is the probability of forecast error larger than 30 nm and less than 60 nm which we have taken to be Weibull distributed with parameters estimated from historical data. So, we have:

\[
P_{\text{ase}} = P_{\text{ase}} \cdot P_{\text{de}} \cdot P_{\text{omde}} (30 < \text{Error} < 60) = 0.2 \cdot \frac{1}{3} \cdot 0.105 = 0.007
\]

Table 3.1 Weighting Functions for Each Section

<table>
<thead>
<tr>
<th>Interval</th>
<th>Ahead</th>
<th>Left</th>
<th>Right</th>
<th>Behind</th>
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<td>12</td>
<td>0.21</td>
<td>0.31</td>
<td>0.24</td>
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<td>0.15</td>
<td>0.25</td>
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<tr>
<td>48</td>
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<td>0.20</td>
<td>0.16</td>
<td>0.24</td>
</tr>
<tr>
<td>72</td>
<td>0.27</td>
<td>0.27</td>
<td>0.16</td>
<td>0.30</td>
</tr>
</tbody>
</table>

The probability that the actual track point is in each different small section can be estimated by the same procedure, and hurricane tracks can be simulated by generating pseudo-random tracks, according to that estimated probability distributions.

Figure 3.4 is the plot of forecast mean distance error vs. forecast time interval. The mean error of 12 hours forecasts is about 50 nm with standard deviation 25 nm, while for 72 hours forecasts, the mean is almost 300 nm with standard derivation almost 200 nm. It is found that the mean hurricane track forecast error increases approximately linearly with
Figure 3.4 Forecast Mean Distance Error vs. Forecast Time Interval
forecast time interval. So does its standard deviation. It is the long "tail" of large forecast errors that place the users of hurricane forecasting at risk. There are large uncertainties associated with hurricane track forecasting and reliability is a very important concept. Simulations are a useful tool to approach the problem.

The same approach can be used to analyze the strength forecast error if changes of hurricane strength need to be included in the simulation.

3.4 Markov-Chain Simulation Model

Sometimes, the simulation is not restricted to one special hurricane and what we are interested in is a long-term period probability. For example, to calculate the annual collision probability of MODU with large surrounding facilities, the hurricane is generated according to Poisson distribution in a one year period and there is no real time hurricane forecast data available. The Markov-Chain simulation model is effective for this purpose.

Based on the data from the MMS (MMS, 1993), the transition probability matrix to describe the probabilities of changes in the storm track directions for Gulf of Mexico hurricanes have been developed. As it was discussed in Section 3.1, the most important hurricane track forecast techniques are persistence and climatology. The Markov model is a combination of these techniques. It is a blend that weights persistence higher early in the forecast and climatology higher later. This empirical blending presumably takes advantage
of the best characteristics of each scheme. The next section provides an introduction to Markov processes.

3.4.1 Introduction to Markov Chains

The state of a system invariably changes with respect to some parameter, for example, time or space. The transition from one state to another as a function of the parameter, or its corresponding transition probability, may generally depend on the prior states. However, if the transition probability depends only on the current state, the process of change may be modeled with the Markov process. If the state space is a countable or finite set, the process is called a Markov Chain.

Consider a system with \( m \) possible states, namely \( 1, 2, \ldots, m \), and changes in state can occur only at discretized values of the parameter; for example, at times \( t_1, t_2, \ldots, t_n \). Let \( X_{n+1} \) denote the state of the system at \( t_{n+1} \). In general, the probability of a future state of the system may depend on its entire history; that is, its conditional probability is:

\[
P(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \ldots, X_1 = x_1)
\]  \hspace{1cm} (3.2)

where \( X_n = x_n, X_{n-1} = x_{n-1}, \ldots, X_1 = x_1 \) represent all previous states of the system. If the future state is governed solely by the present state of the system, that is, the conditional probability, Eq.(3.2) is:

\[
P(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \ldots, X_1 = x_1) = P(X_{n+1} = x_{n+1} | X_n = x_n)
\]  \hspace{1cm} (3.3)
then the process is a Markov chain. For a discrete parameter Markov chain, the transitional probability from state \( i \) at time \( t_m \) to state \( j \) at time \( t_n \) may be denoted by:

\[
p_{ij}(m,n) = p(X_n = j|X_m = i); \quad n > m
\]  

The Markov chain is homogeneous if \( p_{ij}(m,n) \) depends only on the difference \( t_n - t_m \); in this case, we define:

\[
p_{ij}(k) = p(X_n = j|X_0 = i) = p(X_{n-k} = j|X_0 = i) \quad s \geq 0
\]  

as the \( k \)-step transition probability function. Physically, this represents the conditional probability that a homogeneous Markov chain will go from state \( i \) to state \( j \) after \( k \) times stages. This probability can be determined from the one-step transition probabilities, namely \( p_{ij}(1) \) or simply \( p_{ij} \), between all pairs of states in the system. These transition probabilities can be summarized in a matrix for a system with \( m \) states, called the transition probability matrix:

\[
P = \begin{bmatrix}
p_{1,1} & p_{1,2} & \cdots & p_{1,m} \\
p_{2,1} & p_{2,2} & \cdots & p_{2,m} \\
\vdots & \vdots & \ddots & \vdots \\
p_{m,1} & p_{m,2} & \cdots & p_{m,m}
\end{bmatrix}
\]  

As the states of a system are mutually exclusive and collectively exhaustive after each transition, the probabilities in each row add up to 1.0. For a homogeneous discrete Markov chain, the probabilities of the initial states are the only other information needed to define the model behavior at any future time.
3.4.2 State Probabilities

The probabilities of the respective initial states of a system may be denoted by a row matrix:

\[ P(0) = [p_1(0), p_2(0), \ldots, p_n(0)] \]  \hspace{1cm} (3.7)

where \( p_i(0) \) is the probability that the system is initially at state \( i \). In the special case for which the initial state of the system is known, for example, at state \( i \), then \( p_i(0) = 1.0 \) and all other elements in the row matrix \( P(0) \) are zero. After one transition, the probability that the system is in state \( j \) is given by the theorem of total probability as:

\[ P_j(1) = P(X_1 = j) = \sum_i P(X_1 = i)P(X_2 = j|X_1 = i) \]  \hspace{1cm} (3.8)

Hence,

\[ P_j(1) = \sum_i p_i(0)p_{ij} \]  \hspace{1cm} (3.9)

In matrix notation, the single state probabilities become,

\[ P(1) = P(0)P \]  \hspace{1cm} (3.10)

which is also a row matrix.

Similarly, the probability that the system is in state \( j \) after two transitions is given by:

\[ P_j(2) = \sum_i P(X_1 = k)P(X_2 = j|X_1 = k) = \sum_i P_k(1)p_{ij} \]  \hspace{1cm} (3.11)

or in matrix notation,
\[ P(2) = P(1)P = P(0)PP = P(0)P^3 \]  \hspace{1cm} (3.12)

Therefore, by induction, it can be shown that the n-stage state probability matrix is given by:

\[ P(n) = P(n-1)P = P(n-2)PP = \cdots = P(0)P^n \]  \hspace{1cm} (3.13)

### 3.4.3 Hurricane Tracks Modeling in Gulf of Mexico (GOM)

For application to hurricane tracks, the states are defined as different directions of storm tracks. And the transition step size is two hours. As shown in Figure 3.5, and based on the statistical analysis of storm track histories, the plane 0 to 180 degrees is divided into 3 blocks. Then there are 3 possible states \((1, 2, 3)\) and the transition probability matrix \(P\) is \(3 \times 3\):

1. direction 0-75 degrees
2. direction 75-100 degrees
3. direction 100-180 degrees

And from the statistical analysis of storm track histories, we also assume that, within each state of direction, the moving direction has a probabilistic distribution. It is a uniform distribution in state 1 and triangular distribution in state 2 and 3. The distribution functions are shown in Figure 3.6.
Figure 3.5 Definition of Storm Track Transmission State

Figure 3.6 Probability Density Function of CTA in State 1, 2, 3
The transition probabilities are estimated by calculating the times of storm track direction changes from one state to the other based on the database of hurricane track history from MMS (MMS, 1993). The Table 3.2 shows the observed transitions in Gulf of Mexico (Florida to Texas) from 1950 to 1992.

**Table 3.2 Observed Transitions in Gulf of Mexico (1950 - 1992)**

<table>
<thead>
<tr>
<th>From\To</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>99</td>
<td>12</td>
<td>25</td>
<td>136</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>2</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>4</td>
<td>15</td>
<td>33</td>
</tr>
</tbody>
</table>

The $P_{ij}$ values are estimated from Table 3.2 using the formula:

$$ P_{ij} = \frac{a_{ij}}{\sum_j a_{ij}} \quad (3.14) $$

**Table 3.3. $P_{ij}$ Values**

<table>
<thead>
<tr>
<th>From\To</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.73</td>
<td>0.09</td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
<td>0.63</td>
<td>0.12</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>0.43</td>
<td>0.12</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The resulting transition probability matrix is:

$$ P = \begin{bmatrix} 0.73 & 0.09 & 0.18 \\ 0.63 & 0.12 & 0.25 \\ 0.43 & 0.12 & 0.45 \end{bmatrix} \quad (3.15) $$
3.4.4 Steady State Probabilities

We note that the state probabilities starting with two different initial states approach one another as the number of transition stages increases. In fact, the state probabilities will converge to a set of steady-state probabilities \( p^* \), which are independent of the initial states. Therefore, at steady-state condition,

\[
P(n + 1) = P(n) = P^*
\]

Hence,

\[
P(n + 1) = P(n)P
\]

\[
P^* = P^*P
\]

For a Markov chain with \( m \) states, this matrix equation represents a set of simultaneous equations as follows:

\[
[p_1^* \cdots p_m^*] = [p_1^* \cdots p_m^*] \begin{bmatrix}
p_{11} & \cdots & p_{1m} \\
\vdots & \ddots & \vdots \\
p_{m1} & \cdots & p_{mm}
\end{bmatrix}
\]

We can find that Eq.(3.19) contains one degree of freedom. The required constraint to obtain \( P^* \) is:

\[
p_1^* + p_2^* + \cdots + p_m^* = 1.0
\]

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Given a particular state matrix $P_i$, the probabilities of being in the various possible states after $n$ transitions are found from:

$$P_{n+1} = P_i \cdot P^r$$  \hspace{1cm} (3.21)

Using the hurricane route input as $P_i = [1 \hspace{0.5cm} 0 \hspace{0.5cm} 0]$, after every two hours, the probability matrix is:

$$P_1 = \begin{bmatrix} 0.73 & 0.09 & 0.18 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.667 & 0.098 & 0.235 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0.649 & 0.099 & 0.250 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0.643 & 0.101 & 0.256 \end{bmatrix}$$

And the $P^r$ matrix is calculated as:

$$P^r = \begin{bmatrix} 0.643 & 0.101 & 0.256 \end{bmatrix}$$  \hspace{1cm} (3.22)

This implies that the hurricane track has about a 64.3\% probability to change to state 1, about a 10.1\% to state 2 and about a 25.6\% to state 3. Also, it can be seen that the future transition probability are not strongly dependent upon the present state matrix. After only four transitions, the state probability matrix coverages to the steady-state probability matrix.
3.4.5 Hurricane Track Modeling for Texas-Mexico Coast

The hurricane route history statistics for Texas-Mexico coastline is calculated based on MMS hurricane forecast database (MMS, 1993). The results are presented as follows:

Table 3.4 Observed Transitions in Texas-Mexico Coast

<table>
<thead>
<tr>
<th>From\To</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>725</td>
<td>74</td>
<td>135</td>
<td>934</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
<td>22</td>
<td>21</td>
<td>98</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td>19</td>
<td>93</td>
<td>232</td>
</tr>
</tbody>
</table>

Table 3.5 $P_{ij}$ Values

<table>
<thead>
<tr>
<th>From\To</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.78</td>
<td>0.08</td>
<td>0.14</td>
</tr>
<tr>
<td>2</td>
<td>0.56</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>3</td>
<td>0.52</td>
<td>0.08</td>
<td>0.40</td>
</tr>
</tbody>
</table>

The transition probability matrix is:

$$P = \begin{bmatrix} 0.78 & 0.08 & 0.14 \\ 0.56 & 0.22 & 0.22 \\ 0.52 & 0.08 & 0.40 \end{bmatrix}$$  \hspace{1cm} (3.23)

The steady-state $P^*$ matrix is calculated as:

$$P^* = \begin{bmatrix} 0.708 & 0.093 & 0.199 \end{bmatrix}$$  \hspace{1cm} (3.24)
Comparing the transition matrix of GOM (Table 3.3) with that of the Texas-Mexico coast (Table 3.5), it is seen that the two matrixes are close to each other. It is also seen that storms tends to move clockwise in the GOM and more straight in the Texas-Mexico coast.

3.4.6 Examples of Markov Model Hurricane Tracks

In Figure 3.7, there are five hurricane track examples generated by MODUSIM based on Markov-Chain model. Study of a wide range of hurricane characteristics indicates that the simulated tracks are representative of real hurricane tracks.

Example 1

Figure 3.7 Examples of Track Forecast Generated by MCSM
Example 4

Example 5
3.5 Hurricane Strength Forecast Error Simulation Model

The most important input parameters for EVACSIM are the forecast wind strength & wave height data at the site. Statistical analysis is needed to simulate the hurricane strength forecasts.

Since the forecasting strength error is roughly proportional to the actual hurricane strength, the statistical analysis of the forecast strength error is based on the relative percent error, which is defined as:

\[
err\% = \frac{\text{Forecast} - \text{True}}{\text{True}} \times 100\%
\]  

(3.25)

The fit results of error percent on different forecast time intervals are presented in Figure 3.8. Again, goodness-of-fit tests were performed. The goodness-of-fit test results were similar to those of track results. The results show that the normal distribution fits the data best (0.1 significance level) among distribution families used (See Appendix C for other distribution families).

To simulate hurricane strength using 72 hours real time forecast data as the most probable value, simulation data are generated as:

\[
\text{Simulation data} = \frac{\text{Forecast}}{1 - X}
\]  

(3.26)

where \( X \) is has a normal distribution with parameters as given in Figure 3.8.
Figure 3.8 Fit Test Results for Hurricane Strength Forecasting
There is high correlation among forecasts 5 hours apart. Based on analysis of the forecast data, the 6 hours correlation coefficient is found to be 0.8.

3.6 Summary

Historical hurricane track forecast data show that there are large uncertainties associated with hurricane track forecasting. Simulation is a suitable way to include the reliability in hurricane track forecasts. An analytical approach would be very difficult, if not impossible, for such a complicated system.

Existing track forecast models are not suitable for simulation processes since they are all complicated and time consuming. Based on a statistical analysis of hurricane track forecast history data, two track forecast simulation models, track forecast error statistical model (TFESM) and Markov-chain simulation model (MCSM), have been developed.

TFESM generates hurricane track forecasts based on the 72 hour real time hurricane forecasts from hurricane forecast centers, and statistical analysis of historical hurricane track forecast errors from the same hurricane forecast centers. This model is especially useful to generate hurricane track forecasts to perform simulations for a special incoming hurricane. MCSM generates hurricane track forecasts based on the Markov transition probability matrix, which is estimated from hurricane track history statistics. The Markov model is a combination of the most important hurricane track forecast techniques, persistence and climatology. It is a blend that weights persistence higher early in the